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'Oumuamua, the Gömböc and the Pebbles of Mars

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In this talk I will concentrate on two examples from planetary science which made the headlines in recent years to highlight the power and significance of nonlinear geometric partial differential equations (PDEs) explaining puzzles presented by Nature.

In 2012 NASA's Curiosity sent home stunning images of rounded pebbles on Mars [1]. It seemed to be a plausible hypothesis that these pebbles have been rounded while they were carried by an ancient river. However, more evidence was called for. The subsequent results provided by geometric PDE models [2] serve as the most eloquent proof of intense fluvial activity on ancient Mars.

In October 2017 the first-ever interstellar asteroid, 'Oumuamua was sighted. Its hugely elongated shape sparked speculations among leading scientists (among others from Harvard University) that it may be a 'alien' spaceship. However, the results provided by geometric PDE models [3] showed that 'Oumuamua's bizarre shape may be the natural outcome of a multi-hundred-million year abrasion process.

The mathematical theory utilized in these models is rather new, it has been developed towards proving the Poincaré-conjecture, one of the major breakthroughs of the past decade, due to R. Hamilton and G. Perelman. Beyond serving a purely mathematical purpose, the geometric PDEs developed and studied by this group turn out to be a fundamental tool in understanding shape evolution processes in the non-living Nature.

One key link between geometric PDE theory and natural abrasion is a curious object called Gömböc whose existence was conjectured in 1995 by V.I. Arnold. He proposed that convex, homogeneous solids with just N=2 static balance points may exist. Ten years later, based on a constructive proof [4] by P.L. Várkonyi and the author, the first such object was built. In everyday terms, such a body (or pebble) would behave like a weeble toy, and it would always right itself on a flat surface under gravity to one and the same rest position. The Gömböc is quite sensitive and almost never found in Nature, so first it appeared to be a mathematical curiosity. Nevertheless, it became clear that the delicate connection between geometric shapes and the number N of their equilibrium points is an essential indicator of shape evolution [5,6] and it can be used to identify and verify PDE models describing this process. Apparently, in natural abrasion N tends to be reduced and so, in this sense, all shapes approach Gömböc shapes. However, they rarely (if ever) reach that goal. As Sir Michael Berry formulated: "The Gömböc exists in Nature but only as a dream".

References:

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[3] G. Domokos, A.Á. Sipos, G.M. Szabó & P.Várkonyi: Explaining the Elongated Shape of 'Oumuamua by the Eikonal Abrasion Model. Research Notes of the AAS, Volume 1, Number 1.
[4] P.L. Varkonyi and G. Domokos: Mono-monostatic bodies: The answer to Arnold's question. The Mathematical Intelligencer, 28(4) pp 34-38. (2006)
[5] M. Grayson: The heat equation shrinks embedded plane curves to round points. J. Diff.Geometry 26(2) pp 285-314 (1987)
[6] G. Domokos: Monotonicity of spatial critical points evolving under curvature-driven flows. J. Nonlinear Sci. 25(2) 247-275 (2015).

The Gömböc made its debut in 2006 on the cover of the journal Mathematical Intelligencer, 28 years after Rubik's Cube was featured. The New York Times listed the Gömböc among the 70 most interesting inventions of the year 2007 and it also appeared on BBC's "QI" talkshow. In 2010 it was the main exhibit of the Hungarian Pavilion at the World Expo at Shanghai. The Gömböc is the insignia for an internationally recognized prize in contemporary mathematics awarded to young mathematicians: the Stephen Smale Prize. Individual, numbered Gömböc pieces are on permanent exhibit at prestigeous locations worldwide, including the Steklov Mathematical Institute in Moscow (001), University of Cambridge (1209), University of Oxford (2013), Institute Poincaré in Paris (1928), University of Leipzig (1409), Hungarian Academy of Sciences (1825), University of Göttingen (1737), Trinity College, Cambridge (1546), University of KwaZulu-Natal, Durban (1910), University of Auckland (1883,2016), Princeton University (1746) and Windsor Castle (1348).

The Graduate School of Mathematical Sciences at the University of Tokyo put Gömböc 1877 on exhibit in its renowned Model Collection on September 20th 2018.