Random walks in random environments

A random walk in a random environment is a Markov chain in a disordered medium. The standard RWRE is described as follows: The **random environment** (the random medium) $\omega = (p_x)_{x\in\mathbb{Z}^d}$ is given by *random variables* p_x which are defined on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$, and which take values in the set of probability distributions on $\mathbb{Z}^d$, with finite support, say. For a fixed environment $\omega \in \Omega$ the random walk $\eta_0, \eta_1, \ldots$ on $\mathbb{Z}^d$ has the law P_ω defined by

$$P_\omega\left(\eta_0 = i_0, \eta_1 = i_1, \ldots, \eta_n = i_n\right) \stackrel{\text{def}}{=} \delta_{i_0 0} \prod_{k=1}^{n} p_{i_{k-1}}(i_k - i_{k-1}).$$

The so called "quenched" properties of this walk are those which hold for $\mathbb{P}$-almost all realizations of ω. Of interest are also "annealed" properties which refer to the semidirect product $\mathbb{P} \otimes P$.

This model (and many generalizations) have attracted a lot of attention in the past decades. For $d = 1$, the basic works are [10], [8], [9], but there had been many interesting developments. Even the case of dimension 1, when one steps away from the nearest neighbor case, leads to interesting problems, first addressed by Eric Key in [7], and then investigated with a new and powerful approach in [3], [5] with many further developments.

In dimensions $d \geq 3$, there is a celebrated work by Bricmont and Kupiainen [6] which treats perturbatively the case of a random environment with a symmetric distribution. Exit distributions have been obtained in [4], and there are recent improved versions in [1], and [2].

We will discuss the approach introduced in [3] for the quasi-one-dimensional case, explaining also some more recent developments. Then exit distributions in three and more dimensions will be discussed, including the recent extensions to the anisotropic case [2], as well as the basic principles in the delicate two-dimensional case where, under the renormalization procedure, the disorder is only logarithmically contracting.

References

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