



FMSP Lectures

Aurelien Djament

(Nantes/CNRS) (by video conference system)

and

Christine Vespa

(Strasbourg)

Functor categories and stable homology of groups

January 18 (Mon) ~ January 22(Fri) Room 056

Part I: Functor homology and polynomial functors

1- Categories of functors (CV) **January 18 (Mon) 15:00 ~ 16:00**

Categorical properties of functor categories (limits, colimits, abelian category)

Yoneda lemma, projective generators

Tensor products: (classical, external, over the source category). Effect on the projective (eg $P_i \otimes P_j \simeq P_{i+j}$)

Examples : Γ , FI , ab , gr , ... + examples of functors (T^n , S^n , Λ^n ...)

Morita equivalences: (eg Pirashvili's theorem à la Dold-Kan).

2- Polynomial functors (CV) January 18 (Mon) 16:30 ~ 17:30

Setup: Symmetric monoidal source and abelian target

Definition with cross-effect (examples)

Other definitions (eg difference functor)

Quotient categories and [recollement diagram]

examples of $R\text{-mod}$ and gr

Graded exponential functors

3- Homology of functors (2 lectures)

1) Ext and Tor (CV) January 19 (Tue) 13:30 ~ 14:30

Homology of a category

First properties: from Tor to Ext

Example with explicit resolution on gr

2) Methods: (AD 1h30) January 19 (Tue) 16:30 ~ 18:00

Change of source category (adjunction at the source (ex: sum-diagonal), derived Kan extensions)

Multifunctors: Künneth formula

Case of polynomial functors (specific methods: Pirashvili's Lemma)

Use of explicit complexes : Koszul, de Rham+ associated hypercohomology spectral sequences (Franjou-Lannes-Schwartz)

[Examples : gr , ab , Gamma (Hodge decomposition of Pirashvili)]

Part II: Stable homology with twisted coefficient

1- General framework and theorem (AD 2h) January 20 (Wed) 16:00 ~ 18:00

What is stable homology?

-Step 1

Homogeneous category+ examples

Example of symmetric groups

General theorem

-Step 2

the general principle

2- An example: $\text{Aut}(F_n)$ (CV) January 21 (Thu) 15:00 ~ 16:00

Statement for covariant coefficients

Sketch of the proof

Statement for contravariant functors and bifunctors

3- Another example: GL_n (AD 1h30) January 21 (Thu) 16:30 ~ 18:00

Scorichenko criterium

Proof

4- Other applications of functor homology (CV) January 22 (Fri) 15:00 ~ 16:00

Homology of algebras over an operad (Hodge decomposition of Pirashvili)

Polynomial functors with an initial object

Stability results (Randall Williams+Wahl)

[THH and MacLane homology]

[Strict polynomial functors]

Questions and Answers session: (AD+CV) January 22 (Fri) 16:30 ~ 17:30

※関連 URL

Notes of lectures given to Tokyo's university by video conference system in January 2016

<http://www.math.sciences.univ-nantes.fr/~djament/courses-tokyo.html>

FUNCTOR CATEGORIES AND STABLE HOMOLOGY VIA FUNCTOR HOMOLOGY

<http://www-irma.u-strasbg.fr/~vespa/Cours-Tokyo.pdf>