



FMSP Lectures

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Geometric Whitney problem: Reconstruction
of a manifold from a point cloud

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Abstract:

We study the geometric Whitney problem on how a Riemannian manifold (M, g) can be constructed to approximate a metric space (X, d_X) . This problem is closely related to manifold interpolation (or manifold learning) where a smooth n -dimensional surface $S \subset \mathbb{R}^m$, $m > n$ needs to be constructed to approximate a point cloud in \mathbb{R}^m . These questions are encountered in differential geometry, machine learning, and in many inverse problems encountered in applications. The determination of a Riemannian manifold includes the construction of its topology, differentiable structure, and metric.

We give constructive solutions to the above problems. Moreover, we characterize the metric spaces that can be approximated, by Riemannian manifolds with bounded geometry: We give sufficient conditions to ensure that a metric space can be approximated, in the Gromov-Hausdorff or quasi-isometric sense, by a Riemannian

manifold of a fixed dimension and with bounded diameter, sectional curvature, and injectivity radius. Also, we show that similar conditions, with modified values of parameters, are necessary.

Moreover, we characterise the subsets of Euclidean spaces that can be approximated in the Hausdorff metric by submanifolds of a fixed dimension and with bounded principal curvatures and normal injectivity radius.

The above interpolation problems are also studied for unbounded metric sets and manifolds. The results for Riemannian manifolds are based on a generalisation of the Whitney embedding construction where approximative coordinate charts are embedded in \mathbb{R}^m and interpolated to a smooth surface. We also give algorithms that solve the problems for finite data.

The results are done in collaboration with C. Fefferman, S. Ivanov, Y. Kurylev, and H. Narayanan.

References:

[1] C. Fefferman, S. Ivanov, Y. Kurylev, M. Lassas, H. Narayanan: Reconstruction and interpolation of manifolds I: The geometric Whitney problem. ArXiv:1508.00674