



FMSP Lectures

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Introduction to 1-summability and resurgence

October 15 (Thursday) 15:00 ~ 18:00 Room 126

Abstract:

The theories of summability and resurgence deal with the mathematical use of certain divergent power series. The first part of the lecture is an introduction to 1-summability. The definitions rely on the formal Borel transform and the Laplace transform along an arbitrary direction of the complex plane. Given an arc of directions, if a power series is 1-summable in that arc, then one can attach to it a Borel-Laplace sum, i.e. a holomorphic function defined in a large enough sector and asymptotic to that power series in Gevrey sense. The second part is an introduction to Ecalle's resurgence theory. A power series is said to be resurgent when its Borel transform is convergent and has good analytic continuation properties: there may be singularities but they must be isolated. The analysis of these singularities, through the so-called alien calculus, allows one to compare the various Borel-Laplace sums attached to the same resurgent 1-summable series. In the context of analytic difference-or-differential equations, this sheds light on the Stokes phenomenon. A few elementary or classical examples will be considered (the Euler series, the Stirling series, a less known example by Poincaré). Special attention must be devoted to non-linear operations: 1-summable series as well as resurgent series form algebras which are stable by composition. An example of a class of non-linear differential equations giving rise to resurgent solutions will be analyzed. The exposition requires only some familiarity with holomorphic functions of one complex variable.