FMSP Lectures

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Introduction to the Langlands-Rapoport conjecture

May 7 (Monday) - May 11(Friday), 15:00 ～ 17:00 Room 123

Abstract:

In 1970s Langlands envisioned a program to compute the Hasse-Weil zeta functions of Shimura varieties as an alternating product of automorphic L-functions, which in particular implies the meromorphic continuation and functional equation for the zeta functions. In 1987, Langlands and Rapoport formulated a precise and far-reaching conjecture describing the set of points of Shimura varieties modulo p as an essential step towards the goal. The program has been largely carried out by Langlands, Kottwitz, and others for PEL-type Shimura varieties with striking applications to the local and global Langlands correspondences (which in turn led to further applications). We have started to understand the more general Hodge-type and abelian-type cases only recently, thanks to Kisin's work on the Langlands-Rapoport conjecture in the good reduction case. The lecture aims to give a gentle introduction to his seminal paper.

After a brief introduction, the lecture is divided into four parts.

(i) Shimura varieties: We introduce Shimura varieties of Hodge type and abelian type and their integral models.

(ii) Statement of the conjecture: After setting up the language of Galois gerbs, we state the Langlands-Rapoport conjecture.

(iii) Sketch of Kisin's proof: We sketch Kisin's proof of the conjecture for Shimura varieties of Hodge type.

(iv) Counting fixed points: Following forthcoming work of Kisin, Y. Zhu, and the speaker, we explain how to apply the Langlands-Rapoport conjecture to count fixed-points of Hecke-Frobenius correspondences.