Mini-workshop "Recent Trends in Traveling Waves"

Time: January 30, 2015, 10:00-17:20

Place: Graduate School of Mathematical Sciences, University of Tokyo, Room 002
(Near Komaba Todaimae Station of Inokashira Line)

——Program——

10:00〜10:40 Danielle Hilhorst (CNRS / Univ. Paris-Sud)
Front propagation in nonlinear parabolic equations

11:00〜11:40 Thomas Giletti (University of Lorraine)
Speed-up of propagation by a road – the periodically heterogeneous framework

11:50〜12:30 Masaharu Taniguchi (Okayama University)
An $(N-1)$-dimensional convex compact set gives an N-dimensional traveling front in the Allen-Cahn equation

12:30〜14:00 Lunch

14:00〜14:40 Cyrill Muratov (New Jersey Institute of Technology)
A variational approach to supercritical fronts for reaction-diffusion equations in cylinders

14:50〜15:30 Xing Liang (Univ. Science and Technology of China)
The spreading speeds of monostable parabolic equations with free boundary in the spatially and temporally periodic domain

15:50〜16:30 Matthieu Alfaro (University of Montpellier)
Acceleration or not in some nonlocal reaction diffusion equations

16:40〜17:20 Hirokazu Ninomiya (Meiji University)
Traveling waves in two dimensional excitable media

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- The University of Tokyo FMSP program
- JSPS KAKENHI Grant nunber 23244017.

Danielle Hilhorst (CNRS / Univ. Paris-Sud)

"Front propagation in nonlinear parabolic equations"

We study the existence and the stability of travelling waves for nonlinear convection-diffusion equations in one-space dimension. The diffusion coefficient depends on the gradient as in the case of the p-Laplacian and it may be degenerate. Unconditional stability is established with respect to initial data perturbations in L^1. We also prove that the solution converges to ± 1 outside of an interface which moves with constant velocity; our results include both generation and propagation of interface properties. This is joint work with Eduard Feireisl, Hana Petzeltová and Peter Takáč.

Thomas Giletti (University of Lorraine)

"Speed-up of propagation by a road – the periodically heterogeneous framework"

In this talk, I will discuss a new model which was initially proposed by H. Berestycki, J.-M. Roquejoffre and Luca Rossi in order to account for the effects of a road on the propagation of a biological species. Under Fisher-KPP type assumptions on the reproduction rate of the species, they showed that the asymptotic speed of spreading is accelerated when the diffusion on the road exceeds some threshold. I will give a brief overview of some recent developments for this type of models, and introduce some work in progress (in collaboration with L. Monsaingeon and M. Zhou) on the extension of the above result to the heterogeneous framework.

Masaharu Taniguchi (Okayama University)

"An $(N-1)$-dimensional convex compact set gives an N-dimensional traveling front in the Allen-Cahn equation"

We study traveling fronts to the Allen-Cahn equation in $\mathbb{R}^N$ for $N \geq 3$. Let $(N-2)$-dimensional smooth surfaces be the boundaries of compact sets in $\mathbb{R}^{N-1}$, and assume that all principal curvatures are positive everywhere. We define an equivalence relation between them, and prove that there exists a traveling front associated with a given surface and that it is asymptotically stable for given initial perturbation. The associated traveling fronts coincide up to phase transition if and only if the given surfaces satisfy the equivalence relation.

Cyrill Muratov (New Jersey Institute of Technology)

"A variational approach to supercritical fronts for reaction-diffusion equations in cylinders"

In this talk, I will present a variational treatment of the supercritical traveling front solutions for scalar reaction-diffusion equations in infinite cylinders which invade a linearly unstable equilibrium. These equations are known to possess traveling wave solutions connecting an unstable equilibrium to the closest stable equilibrium for all speeds exceeding a critical value. I will show how to construct these fronts by minimizing a suitable functional and show that these are, in fact, the only traveling front solutions and are long-time attractors in the considered problems for sufficiently large speeds and suitable initial data. In addition, I will show that other traveling fronts connecting to the unstable equilibrium may exist in a certain range of the wave speed.

Xing Liang (Univ. Science and Technology of China)

"The spreading speeds of monostable parabolic equations with free boundary in the spatially and temporally periodic domain"

In this talk, we investigate the spreading speed of the following free boundary problem:

$$(*)\begin{cases} u_t - du_{xx} = f(t,x,u), & t>0,\ g(t)<x<h(t),\\ u(t,h(t)) = u(t,g(t)) = 0, & t>0,\\ h'(t) = -\mu u_x(t,h(t)), & t>0,\\ g'(t) = -\mu u_x(t,g(t)), & t>0,\\ u(0,x) = u_0(x) > 0, & g(0)\le x\le h(0), \end{cases}$$

where we assume that the media is space-time periodic in the sense that f is periodic in t,x. We also assume that f is smooth and $f(t,x,0)=0$. We say that the equation is 'monostable' if the equation $u_t - du_{xx} = f(t,x,u), x\in\mathbb{R}$, has precisely two time-space periodic solutions, namely 0 and a positive one $\bar{u}(x,t)$, the latter being stable in the space of periodic continuous functions. Then we can prove that there exist constants $c^* > c_* \ge 0$ such that for any initial data (g_0,h_0,u_0) with $g_0 \ll h_0$ and $u_0 \in H_0^1((g_0,h_0))$ with $u_0\ge 0,\ u_0\not\equiv 0$,

$$\lim_{-c't<x<ct,t\to\infty} |u(x,t)-\bar{u}(x,t)| = 0 \quad \text{if } c<c^* \text{ and } c'<c_*,$$
$$\lim_{x<-c't,t\to\infty} u(x,t) = \lim_{x>ct,t\to\infty} u(x,t) = 0 \quad \text{if } c>c^*, c'>c_*.$$

In proving this result, we extend the general theory of Weinberger [2002 JMB] and that of Liang and Zhao [2010 JFA] on the existence of the spreading speed.

Matthieu Alfaro (University of Montpellier)

"Acceleration or not in some nonlocal reaction diffusion equations"

We are concerned with the speed of propagation of solutions to nonlocal reaction-diffusion equations.

First, we consider a class of problems, referred to as replicator-mutator equations in evolutionary genetics. By some tricky change of unknown function, we compute their solution explicitly. Depending on the tails of the initial data, they can accelerate or become extinct (Joint work with R. Carles).

Then, we consider dipersion kernels with heavy tails and degenerate monostable nonlinearities (weak Allee effect). We investigate the balance between these two effects on the propagation of solutions (traveling waves versus acceleration) (Ongoing work with J. Coville).

Hirokazu Ninomiya (Meiji University)

"Traveling waves in two dimensional excitable media"

In this talk we consider the singular limit problems arising from FitzHugh-Nagumo type model, which is a free boundary problem. The singular limit problems of FHN system have been studied by many authors. However, the dynamics of the free boundary problems have not been well known yet. To understand the dynamics, we first introduce the singular limit problem of the modified FitzHugh-Nagumo model and study the dynamics in one dimensional space. Using this observation, I explain the existence of two types of the two-dimensional traveling waves of this model which include the front and the back.