Seminars on Applied Analysis

Date: March 2 (Thu.) – March 3 (Fri.), 2017
Venue: Room 002, Graduate School of Mathematical Sciences, The Univ. of Tokyo

Program

March 2 (Thu.)

15:30-16:15 Prof. Vladimir Romanov (Sobolev Institute of Mathematic)
Phaseless inverse problems for Schrödinger and Helmholtz equations

16:15-17:00 Prof. Mourad Bellassoued (Université de Tunis El Manar)
Carleman estimates and applications

17:10-17:55 Prof. Yves. Dermenjian (Aix-Marseille Université)
Some applications of Carleman estimates for parabolic operators in an isotropic discontinuous media

March 3 (Fri.)

10:00-10:45 Prof. Eric Soccorsi (Aix-Marseille Université)
On Carleman estimates in unbounded cylindrical quantum domains and application to inverse problems

10:45-11:30 Prof. Adam Kubica (Warsaw University of Technology)
Fractional diffusion equation

11:40-12:25 Prof. Piermarco Cannarsa (University of Rome Tor Vergata)
Invariance for quasi-dissipative systems in Banach spaces

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Phaseless inverse problems for Schrodinger and Helmgoltz equations

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For the Schrödinger equation

$$-\Delta_x u - k^2 u + q(x)u = \delta(x-y), \quad x \in \mathbb{R}^3, \tag{1}$$

with the scattering conditions

$$u\,(x,y,k) = O\left(r^{-1}\right),\ \frac{\partial u}{\partial r} + iku = o(r^{-1}),\ r = |x-y| \to \infty. \tag{2}$$

an inverse problem of recovering the potential $q(x)$ is considered. Let $u(x,y,k)$ be the solution of problem (1)-(2) and $u_0(x,y,k)$ be the solution of the same problem with $q(x) \equiv 0$. Denote the scattered field by $u_{sc}(x,y,k) = u(x,y,k) - u_0(x,y,k)$.We assume that $\operatorname{supp} q(x) \subset B \subset \mathbb{R}^3$, where $B = \{x|\,|x| < R\}$, $R > 0$, with the boundary $S = \{x|\,|x| = R\}$. The following inverse problem is considered: given $f(x,y,k) = |u_{sc}(x,y,k)|$ for $(x,y) \in S \times S$ and $k \geq k_0$, $k_0 > 0$, find $q(x)$ in B. The main result consists in reduction of this problem to the usual tomography problem for all straight lines crossing B.

The second inverse problem is related to the Helmgoltz equation

$$\Delta u + k^2 n^2(x) u = -\delta(x-y), \quad x \in \mathbb{R}^3, \tag{3}$$

where $n(x) = 1$ outside of B. We denote solution of (3) satisfying (2) as $u(x,y,k)$ and by $u_0(x,y,k) = e^{ik|x-y|}/(4\pi|x-y|)$ the incident wave in the homogeneous media with $n \equiv 1$. Let $u_{sc}(x,y,k) = u(x,y,k) - u_0(x,y,k)$ be the scattering wave. Consider the inverse problem similar to the previous one: given $f(x,y,k) = |u_{sc}(x,y,k)|$ for $(x,y) \in S \times S$ and $k \geq k_0$, $k_0 > 0$, find $n(x)$ in B. The main result in this case consists in reduction of this problem to the inverse kinematic problem.

Some applications of Carleman estimates for parabolic operators in an isotropic discontinuous media

Assia Benabdallah, Yves Dermenjian & Masahiro Yamamoto

Abstract

We consider

- the cylinder $\Omega := (0,1) \times (-H, H), \Omega_+ := (0,1) \times (0, H), \Omega_- := (0,1) \times (-H, 0)$
- a real diffusion coefficient a such that the restrictions $a_\pm$ to $\Omega_\pm$ belong to $\mathcal{C}^2(\bar{\Omega}_\pm)$ and $0 < c_{\min} \leq a \leq c_{\max} < +\infty$.
- the positive selfadjoint operator $A = -\nabla \cdot (a\nabla)$ with Dirichlet boundary condition.

So, the discontinuities of the coefficient a are included in the interface $S = (0,1) \times \{0\}$. We prove a Carleman estimate for the heat operator $\partial_t + A$ on $\Omega \times (0, T)$ when the localization of the observation is on the boundary of $\partial\Omega$. Precisely, we prove

Theorem 1. *There exist a weight function $\varphi(x,t) := e^{d(x)-\beta(t-t_0)^2}$ and positive constants C, s_0, λ_0 such that, for $s > s_0$ and $\lambda > \lambda_0$, we have*

$$C \int_{\Omega\times(0,T)} \Big(e^{2s\varphi} (s^3\lambda^4\varphi^3|u|^2 + s\lambda^2\varphi|\nabla u|^2 + \frac{1}{s\varphi}|\partial_t u|^2) \mathrm{d}x\mathrm{d}t$$
$$+ \int_{S\times(0,T)} s\lambda\varphi e^{2s\varphi} \big(|(\nabla u)_{|S_i\times(0,T)}|^2 + (s\lambda)^2\varphi^2|u|^2\big) \mathrm{d}\gamma_S \mathrm{d}t - \int_{\partial\Omega\times(0,T)} e^{2s\varphi} s\lambda\varphi a^2 (\partial_\nu d)(\partial_\nu u)^2 \mathrm{d}\gamma\mathrm{d}t$$
$$\leq \int_{\Omega\times(0,T)} f^2 e^{2s\varphi} \mathrm{d}x\mathrm{d}t,$$

for $u(\cdot, t) \in D(A)$ and $u(\cdot, 0) = u(\cdot, T) = 0$.

This kind of inequality for a parabolic operator is well known when the diffusion coefficient is regular or when the interface S is far from the boundary[1]. Here, the interface is tranverse to the boundary and the weight function φ is not usual for a parabolic operator. Starting from this estimate, we shall develop applications that estimate the restriction of u on some subsets of $\Omega \times (0, T)$ from the knowledge of the normal derivative of u on a part of $\partial\Omega$.

[1]Le Rousseau J. & Robbiano L., Local and global Carleman estimates for parabolic operators with coefficients with jumps at interfaces. Inv. math (2011) 183:245-336.

Fractional diffusion equation

Adam Kubica
Warsaw University of Technology

A problem with the time fractional Caputo derivative and general elliptic operator is studied. We prove the existence of weak and regular solutions and investigate the issue of continuity of solution at initial time.

Invariance for quasi-dissipative systems in Banach spaces

Piermarco Cannarsa
University of Rome Tor Vergata

In a separable Banach space, we study the invariance of a closed set K under the action of the evolution equation associated with a maximal dissipative linear operator perturbed by a quasi-dissipative term. Using the distance to the closed set, we give a general necessary and sufficient condition for the invariance of K. Then, we apply our result to several examples of partial differential equations.