The semiflow of a delay differential equation on its solution manifold

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The lecture surveys work after the turn of the millenium on well-posedness of initial value problems for differential equations with variable delay. The focus is on results which provide continuously differentiable solution operators, so that in case studies ingredients of dynamical systems theory, such as local invariant manifolds or Poincaré return maps, become available. We explain why the familar theory of retarded functional differential equations [1,2,4] fails for equations with variable delay, discuss what has been achieved for the latter, for autonomous and for nonautonomous equations, with delays bounded or unbounded, and address open problems.

[1] Diekmann, O., van Gils, S. A., Verduyn Lunel, S. M., and H. O. Walther, *Delay Equations: Functional-, Complex- and Nonlinear Analysis.* Springer, New York, 1995.

[2] Hale, J. K., and S. M. Verduyn Lunel, *Introduction to Functional Differential Equations.* Springer, New York, 1993.

[3] Hartung, F., Krisztin, T., Walther, H. O., and J. Wu, *Functional differential equations with state-dependent delays: Theory and applications.* In: A. Canada, P. Drabek, A. Fonda (Eds), HANDBOOK OF DIFFERENTIAL EQUATIONS, Ordinary Differential Equations, vol. 3, Elsevier Science B. V., North Holland, Amsterdam, 2006, pp. 435-545.

[4] Hino, Y., Murakami, S., and T. Naito, *Functional Differential Equations with Infinite Delay.* Lecture Notes in Math., vol. 1473, Springer, Berlin, 1991.

[5] Walther, H. O., *The solution manifold and C^1-smoothness of solution operators for differential equations with state dependent delay.* J. Diff. Eqs. 195 (2003) 46-65.

[6] Walther, H. O., *Smoothness properties of semiflows for differential equations with state dependent delay.* J. Math. Sciences 124 (2004) 5193-5207.

[7] Walther, H. O., *Differential equations with locally bounded delay.* J. Diff. Eqs. 252 (2012), 3001-3039.

[8] Walther, H. O., *Evolution systems for differential equations with variable time lags.* DOI 10.1007/s10958-014-2086-6, J. Math. Sciences 202 (2014), 911-933.

[9] Walther, H. O., *Topics in delay differential equations.* Jahresbericht der Deutschen Mathematiker-Vereinigung 116 (2014) 87-114. doi: 101365/s13291-014-0086-6.

[10] Walther, H. O., *Semiflows for differential equations with locally bounded delay on solution manifolds in the space $C^1((-\infty, 0], \mathbb{R}^n)$.* Preprint, 30 pp, 2015.