Shilnikov chaos due to state-dependent delay, by means of the fixed point index

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The lecture begins with a brief introduction into initial value problems for differential equations with state-dependent delays.

Then a result by L. P. Shilnikov is recalled: In 1967 he showed that for a smooth vectorfield on a neighbourhood of the origin in $\mathbb{R}^4$, with complex conjugate pairs of eigenvalues of the linearization in each open halfplane, at unequal distances from the imaginary axis, and with a homoclinic solution along which the intersection of stable and unstable manifolds is minimal, there are infinitely many periodic orbits close to the homoclinic loop.

We present a similar result about shift dynamics close to a homoclinic loop for the differential equation

$$x'(t) = -\alpha\, x(t - d(x_t)),$$

where the only nonlinear ingredient is the state-dependent delay [1]. This is joint work with Bernhard Lani-Wayda.

For $\alpha \approx 5\pi/2$ so that the linear equation

$$y'(t) = -\alpha\, y(t-1)$$

is hyperbolic with 2-dimensional unstable space we *construct* a delay functional $d : C([-2,0],\mathbb{R}) \supset U \to (0,2)$ with $d(\phi) = 1$ on a neighbourhood of $\phi = 0$ so that the nonlinear equation has a solution which is homoclinic to zero, with the minmal intersection property as in Shilnikov's result, and with further regularity properties of the linearization of the semiflow along the homoclinic flowline in the *solution manifold* $X \subset C^1([-2,0],\mathbb{R})$.

Shift dynamics for a translation along flowlines close to the homoclinic loop is then established by means of a general approach which employs the fixed point index and homotopies to maps in finite dimensions. This extends work of P. Zgliczynski from 1996.

[1] B. Lani-Wayda and H. O. Walther, *A Shilnikov phenomenon due to state-dependent delay, by means of the fixed point index.* DOI 10.1007/s10884-014-9420-z, J. Dyn. Diff. Eqs., to appear.

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