Workshop 'New Topics on Stochastic and Quantum Interacting Particle Systems'
10 December 2013

All talks will take place in Room 338, Faculty of Science Building No.1, University of Tokyo

Organisers: Seiji Miyashita (Tokyo) and Makoto Katori (Chuo)

PROGRAMME (version 2)

10:00-10:10 (10 min.) **Seiji MIYASHITA (Univ. of Tokyo)**
Opening address

10:10-11:00 (50 min.) **Makoto KATORI (Chuo Univ.)**
An elliptic extension of Dyson's Brownian motion model

-11:10 (10 min.) discussion

11:10-12:00 (50 min.) **Nizar DEMNI (Univ. de Rennes)**
I. Dunkl processes with positive multiplicities and reflected BM in Weyl chambers

-12:10 (10 min.) discussion

[12:10-13:30 (80 min.) lunch]

13:30-14:00 (30 min.) **Sergio ANDRAUS (Univ. of Tokyo)**
Limiting regimes of interacting particle systems using Dunkl operators

-14:10 (10 min.) discussion

14:10-15:00 (50 min.) **Nizar DEMNI (Univ. de Rennes)**
II. Jacobi process: from the univariate process to the free one

-15:10 (10 min.) discussion

15:10-16:00 (50 min.) **Yutaka MATSUO (Univ. of Tokyo)**
Dunkl operator and degenerate double affine Hecke algebra in supersymmetric gauge theories

-16:10 (10 min.) discussion

[16:10-16:30 (20 min.) coffee break]

16:30-17:00 (30 min.) **Chihiro MATSUI (Univ. of Tokyo)**
Multi-state extension of the asymmetric simple exclusion processes

-17:10 (10 min.) discussion

17:10-17:40 (30 min.) **Takashi MORI (Univ. of Tokyo)**
Thermodynamic limit in long-range interacting systems

-17:50 (10 min.) discussion

[18:30-20:30 A buffet supper will be prepared.]

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ABSTRACTS

Makoto KATORI (Chuo Univ.)

An elliptic extension of Dyson's Brownian motion model

We introduce an elliptic extension of Dyson's Brownian motion model, which is a temporally inhomogeneous stochastic process of noncolliding diffusion particles defined on a circle. Using elliptic determinant evaluations related to the reduced affine root system of type A (Rosengren-Schlosser (2006)), we give a determinantal martingale representation of the process and prove that the process is determinantal. By taking the temporally homogeneous limit of the present elliptic determinantal process, equilibrium and nonequilibrium dynamics of noncolliding diffusion processes are studied.

Nizar DEMNI (Univ. de Rennes)

I. Dunkl processes with positive multiplicities and reflected BM in Weyl chambers

I'll show how one proves that the radial Dunkl process is the unique strong solution of a singular SDE when all the multiplicity values are positive. While a first proof was given by B. Schapira, the one discussed here relies on multivoque SDEs and has the merit to answer a conjecture of Gallardo and Yor about the the finiteness of the total sum of the jumps performed by the Dunkl process. In this respect, I'll also exhibit related unpublished results by D. Lepingle. When the multiplicity function vanishes, we derive a Tanaka type SDE for the reflected BM in a Weyl chamber. I'll give a necessary and sufficient condition ensuring that the boundary process is the sum of local times of real BMs on the walls of the chamber.

II. Jacobi process: from the univariate process to the free one

I'll start with the relation between the Jacobi process and projections of the Brownian motion on spheres. This aspect allows to define the matrix-Jacobi process as the radial part of an upper-left corner of a BM on the unitary group. The eigenvalues of this process are shown to evolve like particles in the Weyl alcove of type BC and is a conditioned process in Doob's sense. I'll give another construction of the eigenvalues process as a limit of a markov chain on the Gelfand-Tsetlin graph. Finally, I'll describe the asymptotics of the moments of the matrix process as the size tends to infinity

Sergio ANDRAUS (Univ. of Tokyo)

Limiting regimes of interacting particle systems using Dunkl operators

We examine the extension of Dyson's Brownian motion and the Wishart-Laguerre processes to real positive values of the parameter beta, which we refer to as interacting Brownian motions and interacting Bessel processes. Using analytical and numerical techniques, we investigate the form of these systems' time-scaled particle distributions in the steady state, showing that they correspond to the eigenvalue distribution of the beta-Hermite and beta-Laguerre ensembles of random matrices, and we give estimates of their relaxation time. We also prove that in the freezing limit (when beta tends to infinity), the particle distribution of these systems becomes a sum of delta functions located at the zeroes of the Hermite and Laguerre polynomials. Our analysis is carried out using Dunkl operators, and in particular, Dunkl's intertwining operators. We obtain our results by deriving previously unknown expressions for the intertwining operators

of type A and B.
[1] SA, M. Katori, S. Miyashita, J. Phys. A 45 395201 (2012)
[2] SA, M. Katori, S. Miyashita, arXiv:1309.2733

Yutaka MATSUO (Univ. of Tokyo)

Dunkl operator and degenerate double affine Hecke algebra in supersymmetric gauge theories

We review the the role of Dunkl operators and the symmetry (degenerate double affine Hecke algebra) in the instanton moduli space of supersymmetric gauge theory. Recently it reveals an interesting relation between the partition function of $4D$ gauge theories and $2D$ integrable models.

Chihiro MATSUI (Univ. of Tokyo)

Multi-state extension of the asymmetric simple exclusion processes

There are few far-from-equilibrium systems which are exactly solvable. One of those few examples is the asymmetric simple exclusion process, the one-dimensional stochastic process with volume exclusion. We consider the multi-state extension of the asymmetric simple exclusion process based on the fact that the Markov matrix of this process satisfies the algebraic relations of the Temperley-Lieb algebra. Besides the steady states, we derive the exact expressions of particle-density profiles and particle currents on the steady states under the closed boundary conditions.

Takashi MORI (Univ. of Tokyo)

Thermodynamic limit in long-range interacting systems

Existence of thermodynamic limit is a fundamental result of statistical mechanics in short-range interacting systems. Its conventional proof [1] is valid only for short-range interacting systems, so giving the proof of existence of nontrivial thermodynamic limit for long-range interacting systems would be important and interesting. I will explain the proof briefly and derive the useful variational expressions of the entropy and the free energy.
[1] D. Ruelle, "Statistical Mechanics: Rigorous Results"