ELASTIC WAVES IN STRONGLY HETEROGENEOUS PLATES

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We are interested in the band-gaps phenomenon (or forbidden bands), i.e. intervals of frequency in which there is no propagation of elastic waves; it is well known that they only appear in periodic composites. Let us denote $\varepsilon > 0$ a small parameter which represents the size of each elementary elastic inclusion embedded in an elastic matrix. When the composite is made of microstructures whose characteristics (mass density and elasticity tensor) are of the same order of magnitude in terms of ε, the limit homogeneous material, which is obtained when ε tends to zero, does not present any band gaps. In order to retain band gaps also in the limit it is necessary to consider “strongly” heterogeneous materials, i.e. materials whose characteristics depend upon powers of ε.

In the general framework of linearized three-dimensional elasticity we have shown that for “strongly” heterogeneous composites, the limit homogeneous structure presents when ε tends to zero, for some wavelengths, a negative “mass density” tensor. In other words this proves the existence of acoustic band-gaps.

Among all the models describing the mechanical behavior of two-dimensional elastic plates we consider in this talk both the Reissner–Mindlin’s one and the Kirchhoff–Love’s one. For not “very thin” plates the second-order Reissner–Mindlin (R-M) model is preferred since it takes into account the influence of shear associated with cross-section rotation. For thinner plates, the coupled second-order/fourth-order Kirchhoff–Love (K-L) plate model provides an approximation which is more appropriate than the R-M model. The interest of the extension presented here is due to the 2D-3D differences we have to deal with, more precisely either the second order displacement-rotation coupling for the R-M’s model or the second order/fourth order coupling of in-plane/transversal components of the displacement for the K-L’s model. More precisely we will give the expression of the order of magnitude of the mass densities and the elasticity tensors for the matrix and the inclusions and we will show how to compute these band gaps. The theoretical results are illustrated with some numerical simulations.

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