Various effets of oscillations in reaction-diffusion equations

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A celebrated result of Aronson and Weinberger states that solutions of the reaction-diffusion equation

$$\partial_t u = \Delta u + f(u), \quad t > 0,\ x \in \mathbb{R}^N,$$

typically spread with a given speed c^*. More precisely, for typical choices of the nonlinearity f, and for any compactly supported initial datum, the solution satisfies that

$$\forall 0 < c < c^*, \quad u(t, ct) \to 1,$$
$$\forall c > c^*, \quad u(t, ct) \to 0.$$

Moreover, we emphasize that the speed c^* does not depend on the choice of any such initial datum. In this talk, I will discuss how sensitive this result is to its assumptions, and more specifically to the introduction of some oscillations in various parameters.

In a first part, we will consider a KPP type equation with some spatial heterogeneity. Namely, the reaction $f(x, u)$ depends on the spatial variable and satisfies

$$0 < f(x, u) \leq \partial_u f(x, 0)u, \quad 0 < u < 1,\ x \in \mathbb{R}^N.$$

We will provide several examples inspired from the periodic case where the asymptotic speed may or may not vary throughout the propagation. These examples will highlight the difficulty of characterizing the occurrence of spreading with constant speed. In a second part, we look at an ignition type reaction-diffusion equation, i.e.

$$f(u) = 0, \quad u \leq \theta.$$

Due to the fact that reaction does not occur below the ignition threshold θ, it is natural to expect some spatial propagation even when the initial datum does not decay at infinity. After presenting some results from the literature, we will describe how the speed depends non trivially on oscillations of the initial datum above the ignition threshold.

While this talk will be focused on examples (issued from several collaborations with Jimmy Garnier, François Hamel and Grégoire Nadin), we will raise some new questions and try to provide a better understanding of the dynamics of reaction-diffusions equations in a general framework.