Mathematical analysis of a three component chemotactic system

by

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We consider an initial-boundary value problem describing the formation of colony patterns of bacteria *Escherichia coli.* This model consists of reaction-diffusion equations coupled with the Keller-Segel system from the chemotaxis theory in the space-time domain $\Omega \times (0, \infty)$, where Ω is a bounded domain of $\mathbb{R}^d$ with a sufficiently smooth boundary $\partial\Omega$. More precisely, we study the problem

$$
\begin{aligned}
&u_t = \Delta u - \nabla \cdot (u \nabla \chi(c)) + g(u)nu - b(n)u \ \text{ in } \Omega \times (0, \infty) \\
&c_t = d_c \Delta c + \alpha u - \beta c \ \text{ in } \Omega \times (0, \infty), \\
&n_t = d_n \Delta n - \gamma g(u) nu \ \text{ in } \Omega \times (0, \infty), \\
&w_t = b(n)u \ \text{ in } \Omega \times (0, \infty).
\end{aligned} \tag{0.1}
$$

We supplement these equations with the Neumann boundary conditions

$$
\frac{\partial u}{\partial \nu} = \frac{\partial c}{\partial \nu} = \frac{\partial n}{\partial \nu} = 0 \qquad \text{for} \quad x \in \partial\Omega \quad \text{and} \quad t > 0 \tag{0.2}
$$

as well as with nonnegative initial data

$$
u(x,0) = u_0(x), \ c(x,0) = c_0(x), \ n(x,0) = n_0(x), \ w(x,0) = w_0(x) \quad \text{for} \quad x \in \Omega. \tag{0.3}
$$

Here $u(x,t)$ denotes the density of active bacteria, $w(x,t)$ the density of inactive bacteria, $n(x,t)$ the density of nutrient and $c(x,t)$ the concentration of chemoattractant. We remark that the first three equations are closed for u, c and n and that if these are solved, then w can be obtained from u and n.

We answer questions about the global in time existence of solutions as well as on their large time behavior. Moreover, we show that the solutions of a related model may blow up in finite time.

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