CLASSIFICATION OF BIFURCATION DIAGRAMS FOR SUPERCRITICAL ELLIPTIC DIRICHLET PROBLEMS IN A BALL

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Let $B \subset \mathbb{R}^N$, $N \geq 3$, be a unit ball. We study the shape of the bifurcation curve of the positive solutions to the supercritical elliptic equation in B

$$\begin{cases} \Delta u + \lambda f(u) = 0 & \text{in} \quad B, \\ u = 0 & \text{on} \quad \partial B, \\ u > 0 & \text{in} \quad B, \end{cases}$$

where $f(u) = u^p + g(u)$, $p > p_S := (N+2)/(N-2)$, and $g(u)$ is a lower order term. This problem has a singular solution (λ^*, u^*). In this talk we classify the bifurcation curves with the number of the turning points T. The Joseph-Lundgren exponent

$$p_{JL} := \begin{cases} 1 + \dfrac{4}{N - 4 - 2\sqrt{N-1}} & (N \geq 11), \\ \infty & (2 \leq N \leq 10) \end{cases}$$

plays an important role. We prove the following: Let $m(u^*)$ denote the Morse index of u^*. If $p_S < p < p_{JL}$, then $m(u^*) = \infty$ and $T = \infty$. If $p \geq p_{JL}$, then $m(u^*) < \infty$. In this case, if $m(u^*) = 0$, then $T = 0$, and hence a classical positive solution is unique when it exists. If $m(u^*) \geq 1$, then $m(u^*) \leq T < \infty$. In particular if $N \geq 11$, $p \geq p_{JL}$, $m(u^*) \geq 1$, and u^* is nondegenerate, then the last case occurs. A typical example is $f(u) = (u+1)^p + b(u+1)^q$, $p_S < q < p_{JL} \leq p$. Main tools are the intersection number, a singular Sturm-Liouville theory, and a phase plane analysis. The talk is based on a joint work with Yuki Naito of Ehime University.