Large time behavior, Lyapunov functionals and the rearrangement theory for a nonlocal differential equation

by

Thanh Nam NGUYEN (NIMS)

We consider an initial-boundary value problem for a nonlocal evolution equation of bistable type

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(P^\varepsilon) \begin{cases} u_t = \Delta u + \dfrac{1}{\varepsilon^2}\left(u^2(1-u) - u(1-u)\dfrac{\displaystyle\int_\Omega u^2(1-u)}{\displaystyle\int_\Omega u(1-u)} \right) & \text{in } \Omega \times (0,\infty), \\ \partial_\nu u = 0 & \text{on } \partial\Omega \times (0,\infty), \\ u(x,0) = u_0(x) & x \in \Omega, \end{cases}
$$

and study possible sharp transition layers at the very early stage that the solution of Problem (P^ε) might develop as the parameter ε tends to zero. Here Ω is a bounded domain of $\mathbb{R}^N (N \geq 1)$ with smooth boundary $\partial\Omega$, and ∂_ν is the outer normal derivative to $\partial\Omega$.

It turns out that such transition layers can be investigated via the structure of the ω-limit set of the corresponding nonlocal ordinary differential equation. We prove that for a large class of initial functions, the ω-limit set of the nonlocal ordinary differential equation only contains one element. Furthermore, that element is a step function taking at most two values. The proof bases on the rearrangement theory and the existence of infinitely many Lyapunov functionals.

This is a joint work with Danielle Hilhorst and Philippe Laurençot.