PROFILES FOR BOUNDED SOLUTIONS OF DISPERSIVE EQUATIONS, WITH APPLICATIONS TO ENERGY-CRITICAL WAVE AND SCHRÖDINGER EQUATIONS

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1. Introduction

This article presents a new compactness argument to describe the asymptotics of bounded solutions of focusing nonlinear dispersive equations. We will mainly consider the energy-critical wave equation in space dimension $N \in \{3,4,5\}$, for which our results are more complete:

$$\begin{cases} \partial_t^2 u - \Delta u - |u|^{\frac{4}{N-2}} u = 0, \quad (t,x) \in I \times \mathbb{R}^N \\ u_{\restriction t=0} = u_0 \in \dot{H}^1, \quad \partial_t u_{\restriction t=0} = u_1 \in L^2, \end{cases} \tag{1.1}$$

where I is an interval ($0 \in I$), u is real-valued, $\dot{H}^1 := \dot{H}^1(\mathbb{R}^N)$, and $L^2 := L^2(\mathbb{R}^N)$.

We will also give a consequence of our method for solutions of the energy-critical nonlinear Schrödinger equation (NLS):

$$\begin{cases} i\partial_t u + \Delta u = -|u|^{\frac{4}{N-2}} u, \\ u_{\restriction t=0} = u_0 \in \dot{H}^1. \end{cases} \tag{1.2}$$

The equation (1.1) is locally well-posed in $\dot{H}^1 \times L^2$. If u is a solution, we will denote by $(T_-(u), T_+(u))$ its maximal interval of existence. On $(T_-(u), T_+(u))$, the following two quantities

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