

# SOLUTIONS OF THE FOCUSING NONRADIAL CRITICAL WAVE EQUATION WITH THE COMPACTNESS PROPERTY

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ABSTRACT. Consider the focusing energy-critical wave equation in space dimension 3, 4 or 5. In a previous paper, we proved that any solution which is bounded in the energy space converges, along a sequence of times and in some weak sense, to a solution *with the compactness property*, that is a solution whose trajectory stays in a compact subset of the energy space up to space translation and scaling. It is conjectured that the only solutions with the compactness property are stationary solutions and solitary waves that are Lorentz transforms of the former. In this note we prove this conjecture with an additional non-degeneracy assumption related to the invariances of the elliptic equation satisfied by stationary solutions. The proof uses a standard monotonicity formula, modulation theory, and a new channel of energy argument which is independent of the space dimension.

## CONTENTS

1. Introduction	2
2. Preliminaries on well-posedness	6
3. Properties of stationary solutions	10
4. Proof of the main result	26
5. Convergence to a stationary solution by modulation theory	33
6. Lorentz transformation	45
Appendix A. Estimates on modulated functions	56
Appendix B. Nonexistence of solutions converging fast to a stationary solution	58
Appendix C. Choice of the translation parameter	59
Appendix D. Some space-time estimates	60
References	61

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